# MISSPECIFIED EXPECTATIONS AMONG PROFESSIONAL FORECASTERS: ONLINE APPENDIX

Julio L. Ortiz
Federal Reserve Board

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## Appendix A SPF Sample

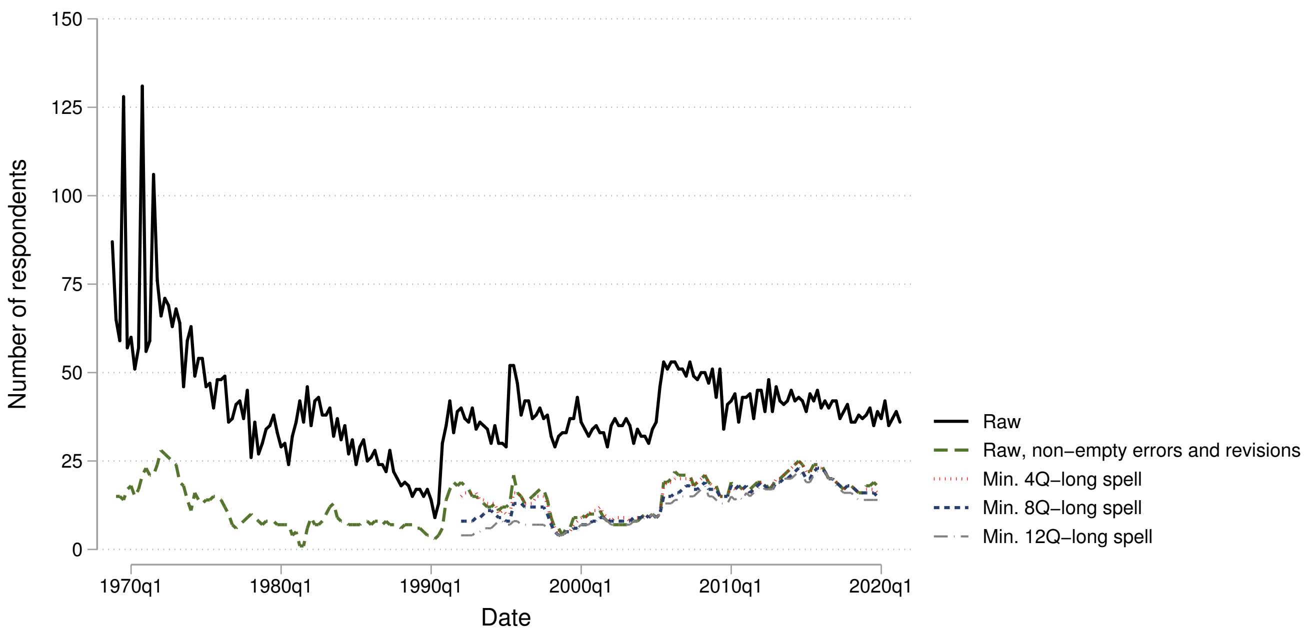
For my analysis, I use data from the Survey of Professional Forecasters website:
https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters.
I collect real GDP forecasts for my baseline results, but also collect other variables listed in the following section. Since forecasters issue predictions for all variables in levels, some forecasts in the dataset must be transformed into growth rates before utilizing them in the analysis. For these variables, such as real GDP, I construct the annualized one-quarter ahead forecast from the forecasted levels, $f^i_{t|t}$ as follows:

$$\widehat{x}^i_{t+1|t} = \left[ \left( \frac{f^i_{t+1|t}}{f^i_{t|t}} \right)^4 - 1 \right] \times 100.$$

Other variables are already measured in growth rates such as CPI and do not require such a transformation.

The SPF is an unbalanced panel. Since forecasters can enter and exit the survey, the number of respondents varies over time. Figure A1 plots the number of respondents over time at different stages of data cleaning for my real GDP growth sample. The solid black line plots the number of respondents in each quarter in the raw data. The dashed green line plots the number of respondents in each quarter conditional on observing a one-quarter ahead forecast error and a one-quarter ahead forecast revision. This requires that a forecaster issue a one-quarter ahead forecast today and have issued a two-quarter ahead forecast in the previous quarter, the latter of which is necessary to construct the forecast revision, $\widehat{x}^i_{t+1|t} - \widehat{x}^i_{t+1|t-1}$. We see that this added restriction reduces the number of respondents in the survey, although the number of respondents is more stable over time.

Figure A1: Respondents Over Time




Note: The figure plots the number of respondents in the SPF issuing real GDP growth forecasts across a range of different sample restrictions. My baseline specification imposes a minimum eight-quarter-long spell for each respondent, leaving me with 77 unique forecasters from 1992Q1 to 2019Q4. The four-quarter- and 12-quarter-long spell requirements are alternative specifications that I consider in Appendix C.

The remaining lines in Figure A1 reflect increasingly more stringent sample restrictions. All of these other three time series start in 1992Q1 which is the selected date in which I begin my baseline sample. The dotted red line reflects the number of respondents when also requiring each respondent to have issued a minimum four-quarter-long string of forecasts. The short dashed navy blue line, which reflects my baseline sample, imposes an eight-quarter-long spell requirement. Finally, the gray dashed-dotted line imposes a 12-quarter-long spell requirement for each respondent. Overall, we see that these requirements impose minor additional restrictions on the number of respondents observed in the sample.

# Appendix B Estimation

In this section I detail the steps taken to estimate the models via MLE.

## B.1 Maximum Likelihood Estimation

Across the different models, we have the following state space set up:

$$\begin{aligned}\mathbf{\epsilon_{it}} &= \mathbf{A}\mathbf{\epsilon_{it-1}} + \mathbf{B}\mathbf{\eta_{it}} \\ \mathbf{z_{it}} &= \mathbf{C}\mathbf{\epsilon_{it}},\end{aligned}$$

Where the latent state, $\epsilon_{it}$ is indexed by forecaster $i$ and date $t$. This state vector includes the components of the macroeconomic variable which include the unobserved state, $x_t$ and its innovations, $w_t$. In addition, this vector includes the unobserved Bayesian forecasts, $x^i_{t|t}$ and $x^i_{t|t-1}$ as well as their consensus analogs. The matrix $\mathbf{A}$ is the transition matrix. The vector $\eta_{it}$ includes the state innovation, $w_t$, and signal noise, $v^i_t$.

The observation vector includes three measurements: individual one-quarter ahead forecast errors, individual one-quarter ahead forecast revisions, and one-quarter ahead consensus forecast errors. I keep only observations for which forecast errors and revisions are both populated, and fix a minimum spell length of eight quarters for which a forecaster must be observed in order to be included.

After stacking all of the forecasters, $i$, we can express the model in a form indexed only by date $t$. The state transition equation is

$$\mathbf{\epsilon_t} = \mathbf{T}\mathbf{\epsilon_{t-1}} + \mathbf{D}\mathbf{u_t}$$

where $\epsilon_{\mathbf{t}} = \begin{pmatrix} \epsilon_{\mathbf{1t}} \\ \epsilon_{\mathbf{2t}} \\ \vdots \\ \epsilon_{\mathbf{nt}} \end{pmatrix}$, $\mathbf{T} = \mathbf{I}_n \otimes \mathbf{A}$, and $\mathbf{D} = \mathbf{I}_n \otimes \mathbf{B}$, and $\mathbf{u_t} = \begin{pmatrix} \mathbf{u_{1t}} \\ \mathbf{u_{2t}} \\ \vdots \\ \mathbf{u}_{nt} \end{pmatrix}$.

The observation equation is:

$$\mathbf{z_t} = \mathbf{M_t} \mathbf{W} \epsilon_{\mathbf{t}}$$

where: $\mathbf{z_t} = \begin{pmatrix} \mathbf{z_{1t}} \\ \mathbf{z_{2t}} \\ \vdots \\ \mathbf{z_{nt}} \end{pmatrix}$, and $\mathbf{W} = \mathbf{I}_n \otimes \mathbf{C}$.

The matrix $\mathbf{M_t}$ is a time-varying $3n_t \times 3n$ matrix, where $n_t$ is the number of forecasters observed at time $t$. This matrix allows me to account for the unbalanced nature of the SPF panel data.

Defining $\mathbf{W_t} = \mathbf{M_t}\mathbf{W}$, the Kalman filter equations are:

$$\begin{aligned}
\mathbf{F_t} &= \mathbf{W_t} \mathbf{P_{t|t-1}} \mathbf{W_t'} \\
\mathbf{K_t} &= \mathbf{P_{t|t-1}} \mathbf{W_t'} \mathbf{F_t^{-1}} \\
\mathbf{z_t^*} &= \mathbf{M_t z_t} - \mathbf{W_t z_{t|t-1}} \\
\mathbf{z_{t+1|t}} &= \mathbf{T}(\mathbf{z_{t|t-1}} + \mathbf{K_t z_t^*}) \\
\mathbf{P_{t+1|t}} &= \mathbf{T}((\mathbf{I_n} - \mathbf{K_t W_t})\mathbf{P_{t|t-1}})\mathbf{T}' + \mathbf{Q}
\end{aligned}$$

where $\mathbf{Q} = \mathbf{I_n} \otimes \begin{pmatrix} \sigma_w^2 & 0 & 0 \\ 0 & \sigma_e^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{pmatrix}$.

The log likelihood is therefore

$$LL = -\frac{1}{2}\left( \sum_t n_t \log(2\pi) + \sum_t \log(\det \mathbf{F_t}) + \mathbf{S_{yy}} \right)$$

where

$$\mathbf{S_{yy}} = \sum_t \mathbf{y_t^{*'}} \mathbf{F_t^{-1}} \mathbf{y_t^*}.$$

I estimate the model by constructing and maximizing the likelihood function numerically.

## B.2 State Space Specifications for a Single Forecaster

### Rational Expectations

State:

$$\begin{bmatrix} x_t \\ x_{t-1} \\ w_{t+1} \\ x^i_{t|t} \\ x^i_{t-1|t} \\ x^i_{t|t-1} \\ x^i_{t-1|t-1} \\ \overline{x}_{t|t} \\ \overline{x}_{t-1|t} \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \kappa_1\rho_1 & \kappa_1\rho_2 & 0 & (1-\kappa_1)\rho_1 & (1-\kappa_1)\rho_2 & 0 & 0 & 0 & 0 \\ \kappa_2\rho_1 & \kappa_2\rho_2 & 0 & 1-\kappa_2\rho_1 & -\kappa_2\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_1 & \rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \kappa_1\rho_1 & \kappa_1\rho_2 & 0 & 0 & 0 & 0 & 0 & (1-\kappa_1)\rho_1 & (1-\kappa_1)\rho_2 \\ \kappa_2\rho_1 & \kappa_2\rho_2 & 0 & 0 & 0 & 0 & 0 & 1-\kappa_2\rho_1 & -\kappa_2\rho_2 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ w_t \\ x^i_{t-1|t-1} \\ x^i_{t-2|t-1} \\ x^i_{t-1|t-2} \\ x^i_{t-2|t-2} \\ \overline{x}_{t-1|t-1} \\ \overline{x}_{t-2|t-1} \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \kappa_1 & \kappa_1 \\ 0 & \kappa_2 & \kappa_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \kappa_1 & 0 \\ 0 & \kappa_2 & 0 \end{bmatrix} \begin{bmatrix} w_{t+1} \\ w_t \\ v^i_t \end{bmatrix}$$

Measurement:

$$\begin{bmatrix} x_{t+1} - \widehat{x}^i_{t+1|t} \\ \widehat{x}^i_{t+1|t} - \widehat{x}^i_{t+1|t-1} \\ x_{t+1} - x_{t+1|t} \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & 1 & -\rho_1 & -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_1 & \rho_2 & -\rho_1 & -\rho_2 & 0 & 0 \\ \rho_1 & \rho_2 & 1 & 0 & 0 & 0 & 0 & -\rho_1 & -\rho_2 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \\ w_{t+1} \\ x^i_{t|t} \\ x^i_{t-1|t} \\ x^i_{t|t-1} \\ x^i_{t-1|t-1} \\ \overline{x}_{t|t} \\ \overline{x}_{t-1|t} \end{bmatrix}$$

where $\begin{bmatrix} w_{t+1} \\ w_t \\ v^i_t \end{bmatrix} \sim N(\overline{\mu}, \Sigma)$, with $\overline{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \sigma^2_w & 0 & 0 \\ 0 & \sigma^2_w & 0 \\ 0 & 0 & \sigma^2_v \end{bmatrix}$.

**Overconfidence:** The overconfidence model is similar, with the forecaster's filtering problem incorporating perceived signal noise, $\alpha_v\sigma_v$, which leads to distorted gains $\{\widehat{\kappa}_1, \widehat{\kappa}_2\}$.

## Diagnostic Expectations

State:

$$
\begin{bmatrix} x_t \\ x_{t-1} \\ w_{t+1} \\ x^i_{t|t} \\ x^i_{t-1|t} \\ x^i_{t|t-1} \\ x^i_{t-1|t-1} \\ x^i_{t-2|t-1} \\ x^i_{t-1|t-2} \\ x^i_{t-2|t-2} \\ \overline{x}_{t|t} \\ \overline{x}_{t-1|t} \\ \overline{x}_{t|t-1} \\ \overline{x}_{t-1|t-1} \end{bmatrix}
=
\begin{bmatrix}
\rho_1 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_1\rho_1 & \kappa_1\rho_2 & 0 & (1-\kappa_1)\rho_1 & (1-\kappa_1)\rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_2\rho_1 & \kappa_2\rho_2 & 0 & 1-\kappa_2\rho_1 & -\kappa_2\rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_1 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\kappa_1\rho_1 & \kappa_1\rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\kappa_1)\rho_1 & (1-\kappa_1)\rho_2 & 0 & 0 \\
\kappa_2\rho_1 & \kappa_2\rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\kappa_2\rho_1 & -\kappa_2\rho_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 & \rho_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} x_{t-1} \\ x_{t-2} \\ w_t \\ x^i_{t-1|t-1} \\ x^i_{t-2|t-1} \\ x^i_{t-1|t-2} \\ x^i_{t-2|t-2} \\ x^i_{t-3|t-2} \\ x^i_{t-2|t-3} \\ x^i_{t-3|t-3} \\ \overline{x}_{t-1|t-1} \\ \overline{x}_{t-2|t-1} \\ \overline{x}_{t-1|t-2} \\ \overline{x}_{t-2|t-2} \end{bmatrix}
$$

$$
+ \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & \kappa_1 & \kappa_1 \\
0 & \kappa_2 & \kappa_2 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \kappa_1 & 0 \\
0 & \kappa_2 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} w_{t+1} \\ w_t \\ v^i_t \end{bmatrix}
$$

$$\text{where} \begin{bmatrix} w_{t+1} \\ w_t \\ v_t^i \end{bmatrix} \sim N(\overline{\mu}, \Sigma), \text{ with } \overline{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \sigma_w^2 & 0 & 0 \\ 0 & \sigma_w^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix}.$$

Measurement:

$$\begin{bmatrix} x_{t+1} - \widehat{x}^i_{t+1|t} \\ \widehat{x}^i_{t+1|t} - \widehat{x}^i_{t+1|t-1} \\ x_{t+1} - x_{t+1|t} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{12} & c_{1,3} & c_{1,4} & c_{1,5} & c_{1,6} & c_{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{2,4} & c_{2,5} & c_{2,6} & c_{2,7} & c_{2,8} & c_{2,9} & c_{2,10} & 0 & 0 & 0 & 0 \\ c_{3,1} & c_{3,2} & c_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{4,11} & c_{4,12} & c_{4,13} & c_{4,14} \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \\ w_{t+1} \\ x^i_{t|t} \\ x^i_{t-1|t} \\ x^i_{t|t-1} \\ x^i_{t-1|t-1} \\ x^i_{t-2|t-1} \\ x^i_{t-1|t-2} \\ x^i_{t-2|t-2} \\ \overline{x}_{t|t} \\ \overline{x}_{t-1|t} \\ \overline{x}_{t|t-1} \\ \overline{x}_{t-1|t-1} \end{bmatrix}$$

where

$$
\begin{aligned}
c_{1,1} &= \rho_1 \\
c_{1,2} &= \rho_2 \\
c_{1,3} &= 1 \\
c_{1,4} &= -(1+\varphi)\rho_1 \\
c_{1,5} &= -(1+\varphi)\rho_2 \\
c_{1,6} &= \varphi\rho_1 \\
c_{1,7} &= \varphi\rho_2 \\
c_{2,4} &= (1+\varphi)\rho_1 \\
c_{2,5} &= (1+\varphi)\rho_2 \\
c_{2,6} &= -\varphi\rho_1 \\
c_{2,7} &= -(1+\varphi)(\rho_1^2+\rho_2) + \varphi\rho_2 \\
c_{2,8} &= \varphi(\rho_1^2+\rho_2) \\
c_{2,9} &= -(1+\varphi)\rho_1\rho_2 \\
c_{2,10} &= \varphi\rho_1\rho_2 \\
c_{3,1} &= \rho_1 \\
c_{3,2} &= \rho_2 \\
c_{3,3} &= 1 \\
c_{3,11} &= -(1+\varphi)\rho_1 \\
c_{3,12} &= -(1+\varphi)\rho_2 \\
c_{3,13} &= \varphi\rho_1 \\
c_{3,14} &= \varphi\rho_2
\end{aligned}
$$

**Misspecified Expectations**

State:

$$\begin{bmatrix} x_t \\ x_{t-1} \\ w_{t+1} \\ x^i_{t|t} \\ x^i_{t|t-1} \\ \overline{x}_{t|t} \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \kappa_1\rho_1 & \kappa_1\rho_2 & 0 & (1-\kappa_1)\widehat{\rho} & 0 & 0 \\ 0 & 0 & 0 & \widehat{\rho} & 0 & 0 \\ \kappa_1\rho & \kappa_1\rho_2 & 0 & 0 & 0 & (1-\kappa_1)\widehat{\rho} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ w_t \\ x^i_{t-1|t-1} \\ x^i_{t-1|t-2} \\ \overline{x}_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \kappa_1 & \kappa_1 \\ 0 & 0 & 0 \\ 0 & \kappa_1 & 0 \end{bmatrix} \begin{bmatrix} w_{t+1} \\ w_t \\ v^i_t \end{bmatrix}$$

Measurement:

$$\begin{bmatrix} x_{t+1} - \widehat{x}^i_{t+1|t} \\ \widehat{x}^i_{t+1|t} - \widehat{x}^i_{t+1|t-1} \\ x_{t+1} - \widehat{x}^i_{t+1|t} \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & 1 & -\widehat{\rho} & 0 & 0 \\ 0 & 0 & 0 & \widehat{\rho} & -\widehat{\rho} & 0 \\ \rho_1 & \rho_2 & 1 & 0 & 0 & -\widehat{\rho} \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \\ w_t \\ x^i_{t|t} \\ x^i_{t|t-1} \\ \overline{x}_{t|t} \end{bmatrix}$$

where $\begin{bmatrix} w_{t+1} \\ w_t \\ v^i_t \end{bmatrix} \sim N(\overline{\mu}, \Sigma)$, with $\overline{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \sigma_w^2 & 0 & 0 \\ 0 & \sigma_w^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix}$.

# Appendix C Robustness

## C.1 Likelihood Ratio Tests Across Variables

Table C1 reports the test statistics from the Vuong (1989) test across the different macroeconomic variables. For this one-sided test, a test statistic exceeding the critical value indicates a rejection of the null that misspecified expectations is observationally equivalent to a given model. For the majority of macroeconomic variables, misspecified expectations provides the superior fit to the data.

Table C1: Likelihood Ratio Tests

| | Rational Expectations | Overconfident Expectations | Diagnostic Expectations |
|---|---|---|---|
| CPI | 3.871*** | 3.029*** | 3.812*** |
| GDP Deflator | 4.410*** | 4.910*** | 5.387*** |
| Housing starts | 9.520*** | 9.520*** | 9.520*** |
| Industrial production | 0.875 | 1.031 | -0.993 |
| Payroll employment | 0.737 | 0.772 | 0.737 |
| Real consumption expenditures | 4.285*** | 4.295*** | 5.833*** |
| Real federal government spending | 5.543*** | 5.582*** | 1.919** |
| Real GDP | 2.396*** | 1.005 | 0.680 |
| Real nonresidential investment | 1.663** | 1.329 | -0.052 |
| Real residential investment | 3.679*** | 3.359*** | 1.443 |
| Real state and local government spending | 5.233*** | 5.483*** | 5.221*** |
| Unemployment rate | 1.631* | 1.668** | 1.631* |
| 3-month Treasury bill | 6.016*** | 6.016*** | 7.670*** |
| 10-year government bond | 9.252*** | 9.282*** | 9.252 *** |

Note: The table reports the test statistics from the Vuong (1989) test. Each row of the table refers to macroeconomic variable in the SPF, and each column denotes a pairwise comparison between the misspecified expectations model and another model, labeled by column. *** denotes 1% significance, ** denotes 5% significance, and * denotes 10% significance for these one-sided tests.

## C.2 Estimates over Longer Sample Period

Table C2: Baseline Estimates (1968-2019)

| *Panel A: Stage 1 Parameter Estimates* | | | | | |
|---|---|---|---|---|---|
| First order autocorrelation | $\rho_1$ | 0.492 | | | |
| | | (0.062) | | | |
| Second order autocorrelation | $\rho_2$ | -0.005 | | | |
| | | (0.059) | | | |
| Persistent innovation dispersion | $\sigma_w$ | 2.582 | | | |
| | | (0.136) | | | |

| *Panel B: Stage 2 Parameter Estimates* | | | | | |
|---|---|---|---|---|---|
| | | (1) Rational Expectations | (2) Overconfident Expectations | (3) Diagnostic Expectations | (4) Misspecified Expectations |
| Private noise dispersion | $\sigma_v$ | 1.914 | 1.914 | 1.914 | 1.914 |
| | | (0.075) | - | - | - |
| Overconfidence | $\alpha_v$ | | 0.585 | | |
| | | | (0.064) | | |
| Diagnosticity | $\varphi$ | | | 0.219 | |
| | | | | (0.028) | |
| Perceived persistence | $\widehat{\rho}$ | | | | 0.709 |
| | | | | | (0.031) |
| Log likelihood | | -16665 | -16124 | -16231 | -15680 |
| AIC | | 33341 | 32260 | 32475 | 31371 |
| Encompassing weight | | 0.000 | 0.000 | 0.832 | 0.168 |

Note: Panel A reports estimates for the first stage MLE which estimates the parameters governing the fundamental process. Panel B reports the parameter estimates based on the second and third steps of the estimation procedure. Standard errors reported in parentheses. For each model, panel C reports the maximized log likelihood, AIC, and encompassing weight.

## Table C3: Other Macroeconomic Variables (Longer Sample)

| | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
|---|---|---|---|---|
| CPI | 0.000 | 0.000 | 0.142 | 0.858 |
| GDP Deflator | 0.000 | 0.000 | 0.881 | 0.119 |
| Housing starts | 0.056 | 0.000 | 0.056 | 0.888 |
| Industrial production | 0.000 | 0.970 | 0.000 | 0.030 |
| Payroll employment | 0.000 | 1.000 | 0.00 | 0.000 |
| Real consumption expenditures | 0.000 | 0.000 | 0.047 | 0.953 |
| Real federal government spending | 0.000 | 0.000 | 0.068 | 0.932 |
| Real GDP | 0.000 | 0.000 | 0.832 | 0.168 |
| Real nonresidential investment | 0.000 | 0.000 | 0.000 | 1.000 |
| Real residential investment | 0.000 | 0.000 | 1.000 | 0.000 |
| Real state and local government spending | 0.000 | 0.000 | 0.000 | 1.000 |
| Unemployment rate | 0.000 | 0.365 | 0.000 | 0.635 |
| 3-month Treasury bill | 0.000 | 0.000 | 0.000 | 1.000 |
| 10-year government bond | 0.000 | 0.160 | 0.000 | 0.840 |

Note: The table reports encompassing weights for each for 14 macroeconomic variables covered in the SPF.

## C.3 AR(1) Process

I re-estimate the baseline model for real GDP growth forecasts assuming that the data generating process follows an AR(1) rather than an AR(2). In this case, misspecified expectations imply that forecasters fully understand the AR process governing real GDP growth, $x_t = \rho x_{t-1} + w_t$, but they assign the wrong persistence to it, $\widehat{\rho}$ as in Fuster et al. (2012). Table C4 reports the results which show that misspecified expectations outperform the alternatives when considering simpler dynamics.

Table C4: AR(1) Estimates

| | | | | | |
|---|---|---|---|---|---|
| *Panel A: Fundamental Parameters* | | | | | |
| Autocorrelation | $\rho$ | 0.432 | | | |
| | | (0.089) | | | |
| Innovation dispersion | $\sigma_w$ | 1.663 | | | |
| | | (0.109) | | | |
| *Panel B: Information/Bias Parameters* | | | | | |
| | | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
| Private noise dispersion | $\sigma_v$ | 1.528 | 1.528 | 1.528 | 1.528 |
| | | (0.059) | - | - | - |
| Overconfidence | $\alpha_v$ | | 0.721 | | |
| | | | (0.038) | | |
| Diagnosticity | $\varphi$ | | | 4.526 | |
| | | | | (0.579) | |
| Perceived persistence | $\widehat{\rho}$ | | | | 0.564 |
| | | | | | (0.017) |
| *Panel C: Model Selection* | | | | | |
| Log likelihood | | -8285 | -8093 | -14687 | -8050 |
| AIC | | 16578 | 16197 | 29383 | 16109 |
| Encompassing weight | | 0.260 | 0.000 | 0.135 | 0.606 |

Note: Panel A reports estimates for the first stage MLE which estimates the parameters governing the fundamental process. Panel B reports the parameter estimates based on the second and third steps of the estimation procedure. Standard errors reported in parentheses. For each model, panel C reports the maximized log likelihood, AIC, and encompassing weight.

Table C5 reports the encompassing weights for each variable. For most variables, misspecified expectations outperforms the other models, consistent with the baseline AR(2) results.

# Table C5: Other Macroeconomic Variables (AR1)

| | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
|---|---|---|---|---|
| CPI | 0.000 | 0.000 | 0.390 | 0.610 |
| GDP Deflator | 0.000 | 0.388 | 0.000 | 0.612 |
| Housing starts | 0.181 | 0.000 | 0.176 | 0.644 |
| Industrial production | 0.000 | 0.274 | 0.726 | 0.000 |
| Payroll employment | 0.000 | 0.000 | 0.259 | 0.741 |
| Real consumption expenditures | 0.000 | 0.000 | 0.000 | 1.000 |
| Real federal government spending | 0.000 | 0.000 | 0.000 | 1.000 |
| Real GDP | 0.260 | 0.000 | 0.135 | 0.606 |
| Real nonresidential investment | 0.000 | 0.067 | 0.129 | 0.804 |
| Real residential investment | 0.000 | 1.000 | 0.000 | 0.000 |
| Real state and local government spending | 0.000 | 0.000 | 0.162 | 0.838 |
| Unemployment rate | 0.362 | 0.361 | 0.277 | 0.000 |
| 3-month Treasury bill | 0.000 | 0.505 | 0.405 | 0.090 |
| 10-year government bond | 0.000 | 0.465 | 0.494 | 0.041 |

Note: The table reports encompassing weights for each for 14 macroeconomic variables covered in the SPF.

## C.4 Alternative Spell Lengths

Table C6: Real GDP Estimates (Minimum 4-Quarter Spells)

| | | | | | |
|---|---|---|---|---|---|
| *Panel A: Fundamental Parameters* | | | | | |
| First order autocorrelation | $\rho_1$ | 0.434 | | | |
| | | (0.009) | | | |
| Second order autocorrelation | $\rho_2$ | -0.006 | | | |
| | | (0.002) | | | |
| Persistent innovation dispersion | $\sigma_w$ | 1.663 | | | |
| | | (0.012) | | | |
| *Panel B: Information/Bias Parameters* | | | | | |
| | | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
| Private noise dispersion | $\sigma_v$ | 1.467 | 1.467 | 1.467 | 1.467 |
| | | (0.061) | - | - | - |
| Overconfidence | $\alpha_v$ | | 0.683 | | |
| | | | (0.040) | | |
| Diagnosticity | $\varphi$ | | | 0.249 | |
| | | | | (0.021) | |
| Perceived persistence | $\widehat{\rho}$ | | | | 0.589 |
| | | | | | (0.020) |
| *Panel C: Model Selection* | | | | | |
| Log likelihood | | -9534.8 | -9265.1 | -9269.5 | -9116.3 |
| AIC | | 19080 | 18542 | 18551 | 18345 |
| Encompassing weight | | 0.000 | 0.234 | 0.381 | 0.385 |

Note: Panel A reports estimates for the first stage MLE which estimates the parameters governing the fundamental process. Panel B reports the parameter estimates based on the second and third steps of the estimation procedure. Standard errors reported in parentheses. For each model, panel C reports the maximized log likelihood, AIC, and encompassing weight.

# Table C7: All Variables (Minimum 4-Quarter Spells)

| | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
|---|---|---|---|---|
| CPI | 0.000 | 0.000 | 0.135 | 0.865 |
| GDP Deflator | 0.000 | 0.000 | 0.743 | 0.257 |
| Housing starts | 0.241 | 0.000 | 0.000 | 0.759 |
| Industrial production | 0.000 | 0.000 | 1.000 | 0.000 |
| Payroll employment | 0.000 | 0.000 | 0.000 | 1.000 |
| Real consumption expenditures | 0.000 | 0.063 | 0.196 | 0.741 |
| Real federal government spending | 0.000 | 0.000 | 0.000 | 1.000 |
| Real GDP | 0.000 | 0.234 | 0.381 | 0.385 |
| Real nonresidential investment | 0.000 | 0.000 | 0.000 | 1.000 |
| Real residential investment | 0.000 | 0.000 | 0.661 | 0.339 |
| Real state and local government spending | 0.000 | 0.000 | 0.205 | 0.795 |
| Unemployment rate | 0.741 | 0.000 | 0.000 | 0.259 |
| 3-month Treasury bill | 0.000 | 0.313 | 0.000 | 0.687 |
| 10-year government bond | 0.000 | 0.277 | 0.000 | 0.723 |

Note: The table reports encompassing weights for each for 14 macroeconomic variables covered in the SPF.

# Table C8: Real GDP Estimates (Minimum 12-Quarter Spells)

| | | | | | |
|---|---|---|---|---|---|
| *Panel A: Fundamental Parameters* | | | | | |
| First order autocorrelation | $\rho_1$ | 0.434 | | | |
| | | (0.009) | | | |
| Second order autocorrelation | $\rho_2$ | -0.006 | | | |
| | | (0.002) | | | |
| Persistent innovation dispersion | $\sigma_w$ | 1.663 | | | |
| | | (0.012) | | | |
| *Panel B: Information/Bias Parameters* | | | | | |
| | | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
| Private noise dispersion | $\sigma_v$ | 1.508 | 1.508 | 1.508 | 1.508 |
| | | (0.048) | - | - | - |
| Overconfidence | $\alpha_v$ | | 0.726 | | |
| | | | (0.043) | | |
| Diagnosticity | $\varphi$ | | | 0.230 | |
| | | | | (0.027) | |
| Perceived persistence | $\widehat{\rho}$ | | | | 0.559 |
| | | | | | (0.018) |
| *Panel C: Model Selection* | | | | | |
| Log likelihood | | -7192.8 | -7034.0 | -7020.5 | -7008.2 |
| AIC | | 14396 | 14080 | 14053 | 14028 |
| Encompassing weight | | 0.000 | 0.000 | 0.437 | 0.563 |

Note: Panel A reports estimates for the first stage MLE which estimates the parameters governing the fundamental process. Panel B reports the parameter estimates based on the second and third steps of the estimation procedure. Standard errors reported in parentheses. For each model, panel C reports the maximized log likelihood, AIC, and encompassing weight.

# Table C9: All Variables (Minimum 12-Quarter Spells)

| | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
|---|---|---|---|---|
| CPI | 0.000 | 0.000 | 0.163 | 0.837 |
| GDP Deflator | 0.000 | 0.000 | 0.111 | 0.889 |
| Housing starts | 0.232 | 0.000 | 0.000 | 0.768 |
| Industrial production | 0.000 | 0.000 | 1.000 | 0.000 |
| Payroll employment | 0.000 | 0.000 | 0.000 | 1.000 |
| Real consumption expenditures | 0.000 | 0.081 | 0.146 | 0.773 |
| Real federal government spending | 0.000 | 0.000 | 0.000 | 1.000 |
| Real GDP | 0.000 | 0.000 | 0.437 | 0.563 |
| Real nonresidential investment | 0.000 | 0.000 | 0.000 | 1.000 |
| Real residential investment | 0.000 | 0.000 | 0.791 | 0.209 |
| Real state and local government spending | 0.000 | 0.000 | 0.000 | 1.000 |
| Unemployment rate | 0.731 | 0.000 | 0.000 | 0.270 |
| 3-month Treasury bill | 0.000 | 0.250 | 0.000 | 0.750 |
| 10-year government bond | 0.000 | 0.230 | 0.000 | 0.770 |

Note: The table reports encompassing weights for each for 14 macroeconomic variables covered in the SPF.

### C.5 Parameter Estimates Across Variables

## Table C10: Macroeconomic Variable Parameter Estimates

| | $\rho_1$ | $\rho_2$ | $\sigma_w$ |
|---|---|---|---|
| CPI | 0.222 | -0.147 | 1.935 |
| | (0.009) | (0.009) | (0.018) |
| GDP Deflator | 0.129 | 0.125 | 0.871 |
| | (0.009) | (0.009) | (0.032) |
| Housing starts | 1.064 | -0.088 | 0.086 |
| | (0.011) | (0.010) | (0.002) |
| Industrial production | 0.617 | -0.019 | 3.507 |
| | (0.009) | (0.008) | (0.055) |
| Payroll employment | 0.901 | -0.076 | 0.803 |
| | (0.009) | (0.008) | (0.003) |
| Real consumption expenditures | 0.125 | 0.218 | 1.716 |
| | (0.008) | (0.009) | (0.014) |
| Real federal government spending | -0.026 | 0.197 | 6.756 |
| | (0.008) | (0.008) | (0.209) |
| Real GDP | 0.434 | -0.006 | 1.663 |
| | (0.009) | (0.002) | (0.012) |
| Real nonresidential investment | 0.384 | 0.213 | 7.209 |
| | (0.009) | (0.009) | (0.237) |
| Real residential investment | 0.343 | 0.147 | 11.222 |
| | (0.040) | (0.063) | (5.829) |
| Real state and local government spending | 0.153 | 0.260 | 2.184 |
| | (0.008) | (0.008) | (0.038) |
| Unemployment rate | 1.513 | -0.525 | 0.245 |
| | (0.007) | (0.007) | (0.0003) |
| 3-month Treasury bill | 1.602 | -0.627 | 0.282 |
| | (0.005) | (0.004) | (0.0004) |
| 10-year government bond | 1.255 | -0.291 | 0.353 |
| | (0.006) | (0.006) | (0.0006) |

Note: The table reports MLE parameter estimates of real time macroeconomic variables when fitted to an AR(2) process. The final three columns report the estimated first order autocorrelation, second order autocorrelation, and standard deviation of the innovation, respectively. Standard errors are reported in parentheses.

## C.6 Joint Estimation of Signal Precision and Biases

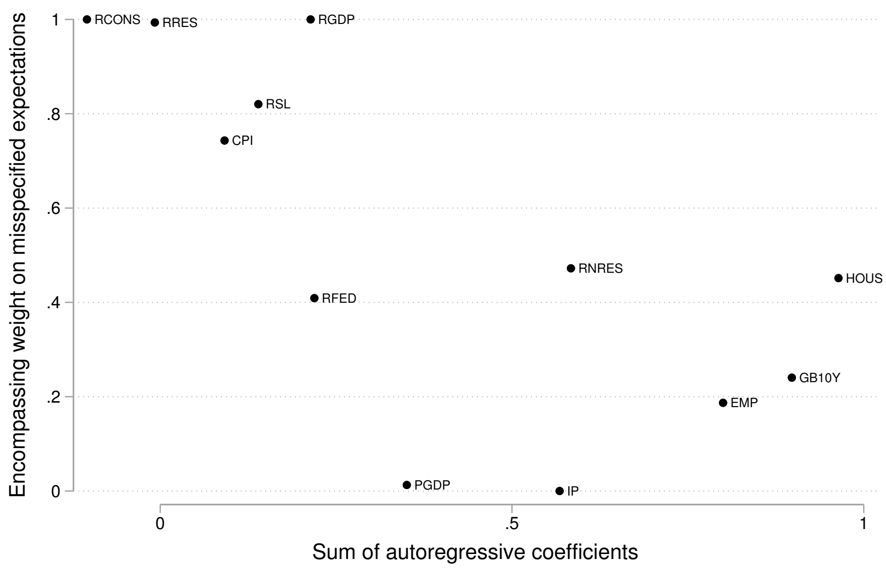
Table C11: Jointly Estimating $\sigma_v$ and Biases

| | Bias parameters estimated only | | Joint $\sigma_v$ and bias estimation | |
|---|---|---|---|---|
| | (1) Diagnostic Expectations | (2) Misspecified Expectations | (3) Diagnostic Expectations | (4) Misspecified Expectations |
| $\sigma_v$ | 1.530 | 1.530 | 1.992 | 2.216 |
| | | | (0.087) | (0.137) |
| $\varphi$ | 0.233 | | 0.364 | |
| | (0.026) | | (0.049) | |
| $\widehat{\rho}$ | | 0.564 | | 0.618 |
| | | (0.018) | | (0.021) |
| Bordalo et al. (2020) coefficient | -0.214 | -0.209 | -0.289 | -0.295 |
| | (0.100) | (0.102) | (0.114) | (0.098) |
| Coibion and Gorodnichenko (2015) coefficient | 0.402 | 0.373 | 0.630 | 0.706 |
| | (0.210) | (0.188) | (0.251) | (0.241) |

Note: Columns (1) and (2) report the point estimates for $\sigma_v$, $\varphi$, and $\widehat{\rho}$ from Table 2 in the main text. Columns (3) and (4) report the point estimates for $\sigma_v$, $\varphi$, and $\widehat{\rho}$ based on a joint two-parameter estimation of the diagnostic expectations and misspecified expectations models. The final rows report the simulated Bordalo et al. (2020) and Coibion and Gorodnichenko (2015) coefficients. Standard errors and standard deviations are reported in parentheses.

## C.7 Misspecified Expectations Fits Less Persistent Series (Out-of-Sample)

Figure C1: Encompassing Weight and Persistence

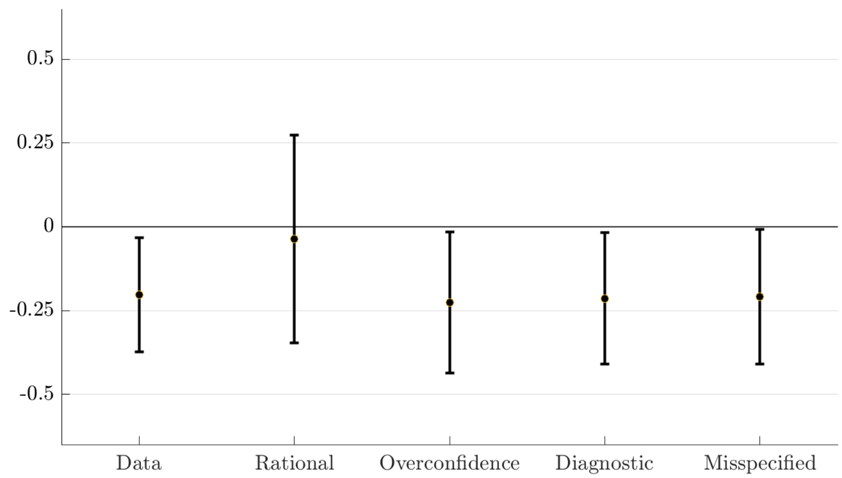



Note: The figure plots the encompassing weight on the misspecified expectations models against the sum of the autoregressive coefficients for each variable, based on the out-of-sample estimation procedure detailed in Section 4.4 of the main text. The figure displays only variables for which forecasters overextrapolate based on the 1992Q1-2005Q4 sub-sample from which the parameters for the out-of-sample procedure are estimated. CPI denotes consumer price index. PGDP denotes GDP deflator. IP denotes industrial production. RCONS denotes real consumption expenditures. RFED denotes real federal government expenditures. RGDP denotes real GDP. RNRES denotes real non-residential investment. RRES denotes real residential investment. RSL denotes real state & local government expenditures. HOUS denotes housing starts. EMP denotes payroll employment. GB10Y denotes the 10-year government bond.

# Appendix D Model Fit to Specific Moments

## D.1 Overreaction

Figure D1: Forecaster-level Errors-on-Revisions Regression




Note: The figure plots the 95% confidence intervals for the individual-level errors-on-revisions regression coefficient in the data as well as the four different models. The model-based coefficients are obtained by simulating 2,000 panels of data for each of the four models. Each model is simulated based on the estimates reported in Table 2.

Bordalo et al. (2020) provide evidence of overreaction in macroeconomic expectations by running the Coibion and Gorodnichenko (2015) errors-on-revisions regression at the forecaster level. This testable prediction has been studied extensively in the literature. The regression specification is:

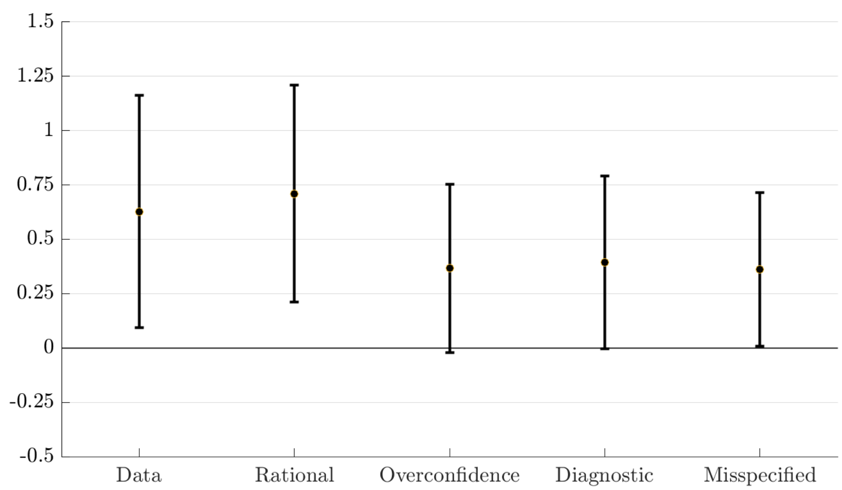
$$x_{t+3} - \widehat{x}^i_{t+3|t} = \beta_0 + \beta_1[\widehat{x}^i_{t+3|t} - \widehat{x}^i_{t+3|t-1}] + \varepsilon^i_t. \tag{1}$$

Estimating $\widehat{\beta}_1 < 0$ implies that an upward ex-ante revision predicts a more negative ex-post error. This negative relation is interpreted as evidence of overreaction to new information.

Figure D1 displays 95% confidence intervals for the $\beta_1$ coefficient in (1) simulated across each of the models, along with the empirical point estimate from the data. As expected, the rational expectations model cannot generate overreactions since forecast errors are orthogonal to anything residing in the forecaster's information set which includes the contemporaneous revision. On the other hand, the other three theories are able to generate negative $\beta_1$ coefficients. Furthermore, these simulated coefficients reside within the 95% confidence interval of the empirical estimate.

## D.2 Underreaction

Figure D2: Consensus Errors-on-Revisions Regression




Note: The figure plots the 95% confidence intervals for the consensus-level errors-on-revisions regression coefficient in the data as well as the four different models. The model-based coefficients are obtained by simulating 2,000 panels of data for each of the four models. Each model is simulated based on the estimates reported in Table 2.

Although individual professional forecasts exhibit overreactions, it is well known that consensus expectations exhibit underreactions (Coibion and Gorodnichenko, 2015). In other words, running (1) at the aggregate level,

$$x_{t+3} - \widehat{x}_{t+3|t} = \alpha_0 + \alpha_1 \big[\widehat{x}_{t+3|t} - \widehat{x}_{t+3|t-1}\big] + \epsilon_t, \tag{2}$$

generally delivers an estimate $\widehat{\alpha}_1 > 0$.

Figure D2 plots the consensus-level analogs to Figure D1. Based on the point estimates, we see that the rational expectations model best matches the data on consensus-level underreactions. This is because the rational model does not feature any scope for overreaction, so the simulated OLS coefficient reflects only the information friction arising from the noisy information environment. Because the other models feature some overreaction at the individual level, and since this overreaction is quantitatively significant, their consensus-level point estimates are lower than the rational model. Nonetheless, the results are significant at the 10% level for all three biased models, and at the 5% level for the misspecified expectations model.[1]

I next turn to examining dynamics, where I find that misspecified expectations is able to generate a sign switch in the impulse response of the aggregate forecast error followed by a gradual convergence to zero, consistent with the data.

## D.3 Overshooting

Angeletos et al. (2020b) documents evidence of delayed overshooting in the medium term, a form of over- and underreaction. This phenomenon can be observed by inspecting the impulse response of the consensus forecast error to a shock. If the impulse response function switches signs, then beliefs are said to exhibit overshooting.

I provide evidence of delayed overshooting for one-quarter ahead real GDP growth forecast errors in my sample. To do so, I follow Angeletos et al. (2020b) by collecting the identified business cycle shocks from Angeletos et al. (2020a).[2] I estimate the impulse response of the one-quarter ahead consensus forecast error in my sample to a positive identified business cycle shock via local projections (Jordà, 2005). I run the following regression across various horizons, $h$,

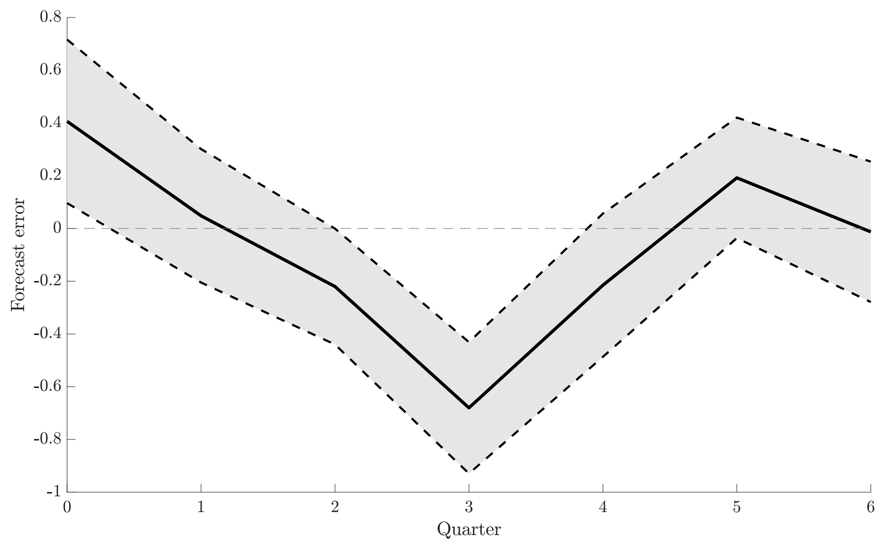
$$\text{Error}_{t+h} = \beta_{0,h} + \beta_{1,h}\text{Shock}_t + \gamma_h' \mathbf{X}_{t-1} + \epsilon_{t+h}, \tag{3}$$

where I specify four lags of the consensus forecast, realized real-time GDP growth, and the identi-

[1]One reason why the biased models generate less underreaction at the consensus-level is because I estimate the biases in the non-rational models only after calibrating the information friction according to the estimates obtained in the rational model. This approach delivers relatively less information rigidity than would be obtained by jointly estimating $\sigma_v$ alongside the bias parameters for each model, as shown in online Appendix C.6.

[2]I utilize the shock that they regard as the "main business cycle shock" which reflects a demand shock, available from their replication files.

# Figure D3: Delayed Overshooting in Real GDP Growth Forecasts




Note: The figure plots the estimated impulse response of the one-quarter ahead consensus forecast error in my sample to an identified business cycle shock from Angeletos et al. (2020a) according to equation (3). Newey-West standard errors are specified and the shaded area reflects 68% confidence intervals.

fied shock as controls.[3]

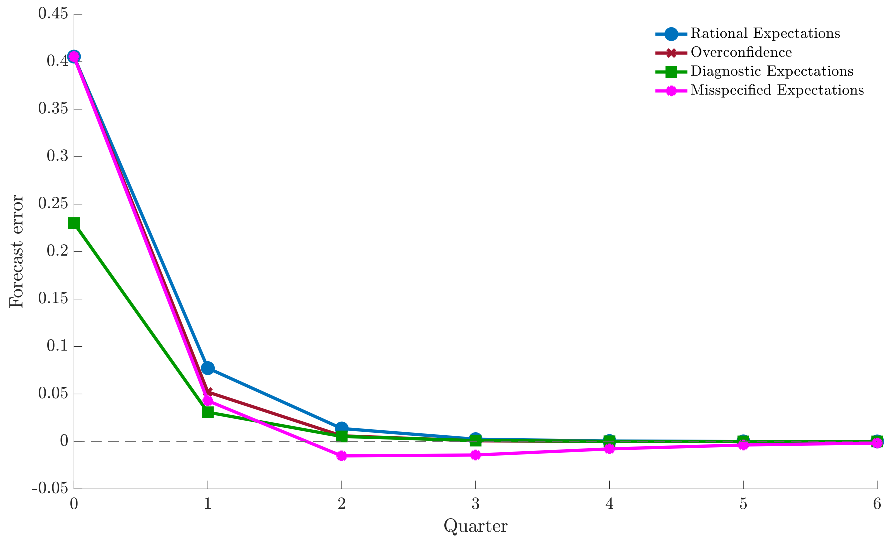
Figure D3 plots the response of the one-quarter ahead average forecast error to a positive demand shock. We see that the forecast error is positive on impact, reflecting the upward surprise in real GDP growth. Thereafter, the one-quarter forecast error declines and begins turning negative two quarters following the shock before reverting back toward zero. The sign-switch in the impulse response of the forecast error observed here is evidence of delayed overshooting.

In principle, all three biased models can reproduce these dynamics given that each model features imperfect information, the source of initial underreaction, and overreactive biases, the source of delayed overreaction. Quantitatively, however, I find that only misspecified expectations produces overshooting.

Figure D4 plots the response of the simulated consensus forecast error to a positive fundamental shock, $w_t > 0$, in each of the four models. I scale the shock to produce a 0.40 percentage point increase in the forecast error on impact, which coincides with the empirical estimate in Figure D3.

[3]These results are robust to specifying alternative lags and controls.

Figure D4: Simulated Overshooting




Note: The figure plots the simulated impulse response of the one-quarter ahead consensus forecast error to a positive shock to $w_t$ scaled to generate a 0.40 percentage point increase in the one-quarter ahead consensus forecast error in the rational expectations model. The impulse responses are obtained by simulating 2,000 panels of forecasts for each of the four models. Each model is simulated based on the estimates reported in Table 2.

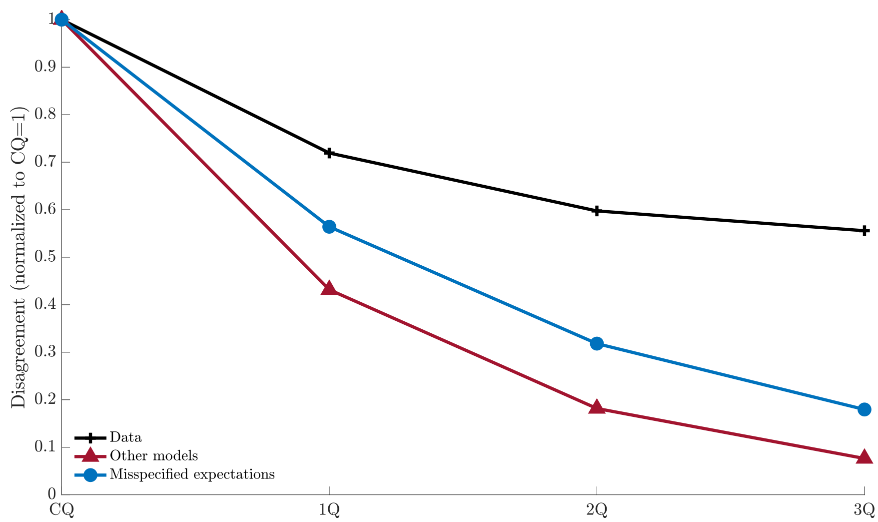
The consensus forecast error in the rational expectations model gradually converges to zero, with the evolution of the forecast error reflecting the rate of learning in the model. The consensus error in the overconfident expectations and diagnostic expectations models feature a stronger convergence to zero, but no sign switch. The misspecified expectations model, on the other hand, does produce a sign switch, with the extent of overshooting peaking two quarters following the shock, after which point the aggregate forecast error converges gradually to zero from below.

Angeletos et al. (2020b) note that overextrapolation is necessary to replicate delayed overshooting dynamics in the medium term. Misspecified expectations, as modeled here, is able to reproduce this pattern better than the other models. Intuitively, this has to do with the fact that forecasters operating under misspecified expectations exhibit overreaction because they misperceive the underlying persistence of the data generating process. As a result, forecast errors are generally longer-lived under misspecified expectations, and, when coupled with a relatively larger $\widehat{\rho}$, forecasters overextrapolate which generates delayed overshooting.

The relatively more persistent forecast errors generated by misspecified expectations can also explain why misspecified expectations better matches forecaster disagreement, which I discuss next.

## D.4 Persistent Disagreement

Figure D5: Persistence in Disagreement Across Horizons




Note: The figure plots a measure of forecaster disagreement at different horizons, where disagreement is defined as the unconditional standard deviation of forecasts. The values of disagreement are normalized so that disagreement in the current quarter is equal to one for all models. 'CQ' standard for 'current quarter', '1Q' stands for one-quarter ahead, '2Q' stands for two-quarters ahead, and '3Q' stands for three-quarters ahead. The model-based estimates are obtained by simulating 2,000 panels of data for each of the four models. Each model is simulated based on the estimates reported in Table 2.

Persistent disagreement is another well-documented puzzle in survey expectations (Andrade et al., 2016; Giacomini et al., 2020; Patton and Timmermann, 2010; Rich and Tracy, 2020). Misspecified expectations outperforms the other models in its ability to generate persistent disagreement across forecasters because it allows forecasters to misstate the degree of mean reversion of the data generating process. Figure D5 plots empirical and model-based forecaster disagreement, which is defined here as forecast dispersion, at different horizons. Each line is normalized to equal one at the current-quarter horizon which facilitates the visual comparison of the persistence in disagreement across models. The black line reflects the data which confirms that disagreement is persistent across horizons. The blue line denotes the misspecified expectations model. The red line reflects the rational expectations, overconfident expectations, and diagnostic expectations models, all of which imply the same evolution in the dispersion of forecasts across horizons.

Because the rational expectations, overconfident expectations, and diagnostic expectations models assume that forecasters know that the data are generated by an AR(2) process, the dispersion of forecasts across horizons will evolve identically according to the AR(2) process. While each of these models implies a different *level* of disagreement, they all imply the same *persistence* in disagreement.[4] On the other hand, because forecasters in the misspecified expectations model misperceive the underlying process, disagreement based on news received today is longer lived. The misspecified expectations model will therefore exhibit a higher dispersion of forecasts over longer horizons relative to the other models.

## D.5 Updating Rules

The forecasters populating the candidate models considered here are fundamentally Bayesian. In this section, I therefore compute the models' implied Kalman gains and compare the weights placed on the private signals across each model and to the data.

Based on the identical information structure assumed across all of the models, forecasters update their predictions as follows:[5]

$$\widehat{x}^i_{t|t} = \widehat{x}^i_{t|t-1} + \widehat{\kappa}_1(y^i_t - \widehat{x}^i_{t|t-1}).$$

This updating equation implies that forecasters place some weight on news, $\kappa_1$, and their prior, $(1-\kappa_1)$,

$$\widehat{x}^i_{t|t} = (1-\widehat{\kappa}_1)\widehat{x}^i_{t|t-1} + \widehat{\kappa}_1 x_t + \widehat{\kappa}_1 v^i_t, \tag{4}$$

[4]When comparing the levels of disagreement, the models have non-overlapping strengths and weaknesses. Diagnostic expectations provides the best fit to the level of disagreement in the current quarter while misspecified expectations provides the best fit to the level of disagreement from the one-quarter ahead horizon onward.

[5]Under rational expectations, $\widehat{\kappa}_1$ coincides with the optimal Kalman gain, $\kappa_1$. Under overconfident expectations and misspecified expectations, $\widehat{\kappa}_1$ is the Kalman gain implied by the perceived signal precision, $\alpha_v$, and perceived persistence, $\widehat{\rho}$, respectively. Under diagnostic expectations, $\widehat{\kappa}_1 = \kappa_1(1+\varphi)$.

Based on this updating rule, I run the following regression to estimate the updating weight,

$$\widehat{x}^i_{t|t} = \beta_0 + \beta_1 \widehat{x}^i_{t|t-1} + \beta_2 x_t + \omega_{it}. \tag{5}$$

This specification projects the current-quarter forecast on the lagged one-quarter ahead forecast and the current realization of the macroeconomic variable. I report empirical and model-based estimates of regression (5) in panel A of Table D1. Based on column (1), there is strong evidence in the data for the updating rule implied by the models considered here. The sum of the estimated coefficients is close to one, and they imply that forecasters place a weight of about 0.35 on new information when updating their expectations.[6]

The other columns of panel A report the simulated model-based regression results. Overall, the rational expectations model provides a better fit to the data, though among the non-rational models, misspecified expectations provides the relatively better fit. We see, however, that forecasters in all estimated models place more weight on new information than on their priors. This is because the Kalman gain in each of these models is relatively high. The Kalman gain is 0.56 in the rational expectations model, 0.71 in the overconfident expectations model, 0.69 in the diagnostic expectations model, and 0.58 in the misspecified expectations model.

The set up that I consider assumes that forecasters only have access to a contemporaneous private signal. Notably, I abstract away from public information. In reality, however, forecasters also observe common signals. A better way to compare the empirical estimates to the model-based regression results would therefore require controlling for common signals by specifying time fixed effects in regression (5). I do so in panel B of Table D1.

By controlling for unobserved time variation, the coefficient in front of the current-quarter realization cannot be estimated since it is absorbed in the time fixed effects. I therefore only report estimates for $\beta_1$. Column (1) reports the empirical estimates under this alternative specification which reveals a meaningful decline in the magnitude of the estimate in front of the lagged one-

[6] A test that the sum of the estimated coefficients is different from one delivers a p-value of 0.76, leading to a failure to reject the null that they sum to one.

## Table D1: Updating Rule Regressions Estimates

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| *Panel A: No time fixed effects* | | | | | |
| | Data | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
| Lagged one-quarter ahead forecast | 0.665*** | 0.437*** | 0.294*** | 0.255*** | 0.418*** |
| | (0.056) | (0.014) | (0.015) | (0.015) | (0.011) |
| Current-quarter realization | 0.348*** | 0.563*** | 0.706*** | 0.694*** | 0.582*** |
| | (0.078) | (0.005) | (0.006) | (0.006) | (0.005) |
| *Panel B: Time fixed effects* | | | | | |
| | Data | Rational Expectations | Overconfident Expectations | Diagnostic Expectations | Misspecified Expectations |
| Lagged one-quarter ahead forecast | 0.421*** | 0.438*** | 0.295 | 0.251*** | 0.419*** |
| | (0.036) | (0.019) | (0.019) | (0.019) | (0.015) |

Note: Each column of the table reports regression results based on (5). Panel A reports the regression results without time fixed effects while panel B specifies time fixed effects. Column (1) reports the empirical regression results, for which I specify Driscoll and Kraay (1998) standard errors in parentheses. The remaining columns specify simulated regression coefficients and standard deviations (in parentheses) based on 2,000 simulated panels of forecasters. * denotes 10% significance, ** denotes 5% significance, and *** denotes 1% significance.

quarter ahead forecast relative to panel A. These results now indicate that forecasters place more importance on new information rather than their priors when updating their expectations. While the empirical results in panel B are different from panel A, the simulated model-based results are nearly identical as expected. As a result, we once again conclude that among the non-rational models, misspecified expectations better matches updating rules in the data.

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